

**Nancy Protheroe**

It involves good teachers,  
an effective math environment,  
and a curriculum that is  
more than a mile wide  
and an inch deep.

Our research-based knowledge about good math instruction; although not as extensive as that focused on reading instruction, has increased in recent years. It now provides a solid base of information for educators to use as they identify mathematics skills students need to develop, as well as teaching strategies and instructional approaches that best support the development of these skills.

When considering content knowledge and skills, it is obvious that schools must look first at the state standards that students are expected to master. However, research comparing math instruction in the U.S. and other countries has pointed to an underlying problem with many of our standards-based systems. Typically, these systems address too many standards for each grade level—encouraging the development of a curriculum that has been characterized as “a mile wide and an inch deep.”

that should be addressed in each grade (see Web resources section on how to access more information about the report). For example, NCTM identifies these math focal points for second grade:

- **Number and Operations.** Developing an understanding of the base-10 numeration system and place-value concepts;
- **Number and Operations and Algebra.** Developing quick recall of addition facts and related subtraction facts and fluency with multidigit addition and subtraction; and
- **Measurement.** Developing an understanding of linear measurement and facility in measuring lengths.

Research indicates that it takes more than a good instructor to teach math. Outlined in this article are recommendations from the National Council of Teachers of Mathematics, as well as other sources, which offer insight on what truly makes math instruction effective.

NCTM suggests that state boards of education and other groups developing standards use the focal points as a "clear organizational model for establishing a mathematics curriculum from pre-kindergarten through grade 8" (NCTM 2006). At the local district and school levels, teacher conversations and staff development could be organized around the focal points. Encouraging teachers from several grades to participate in such a setting ensures discussions are also focused on the linkage of math instruction from grade to grade.

### How It Gets Taught

During the reading wars between proponents of a whole language approach and those favoring skills-based instruction, educators found that a careful and intensive review of research revealed the importance of using a combination of both approaches. Similarly, there are at least two camps prominent in the discussion of how math should be taught. The two teaching approaches have clear differences. In skills-based instruction, teachers focus on developing computational skills and recall of facts. In the second approach, teachers encourage students to explain how they arrived at a solution and to consider more than one way of solving a problem.

Ideally, teachers should strive for a balance between the two approaches. Doug Grouws (2004), recently honored for his long-time contributions to mathematics education with the NCTM Lifetime Achievement Award, talks about this:

*Research suggests it is not necessary for teachers to focus first on skill development and then move on to problem-solving. Both can be done together. Skills can be developed on an as-needed basis, or their development can be supplemented through the use of technology. In fact, there is evidence that if students are initially drilled too much on isolated skills, they have a harder time making sense of them later.*

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What should effective mathematics instruction look like? Shellard and Moyer (2002) identify three critical components: "Teaching for conceptual understanding, developing children's procedural literacy, and promoting strategic competence through meaningful problem-solving investigations."

Also, topics should be presented in a sequence and manner appropriate for the developmental level of the students (Reys *et al.* 1999). Although the rate at which children develop mathematically varies from child to child, NCTM (2001) has developed a general timeline for students' mathematical skills development and instruction identified as appropriate for each level.

According to this timeline:

- From pre-kindergarten through second grade, children develop a mathematical foundation by building beliefs about what mathematics is and what it means to understand and "do" mathematics. Instruction should be provided that helps them understand patterns and measurement and develop a solid understanding of the numeration system.
- Building on the inquisitive nature of children in grades 3 through 5, students should be encouraged to develop and investigate solutions to everyday problems. Instruction should focus on the relationship between such processes as addition and multiplication, and subtraction and division. Students should be introduced to multiplicative reasoning, equivalence, and a variety of methods for computation.

Instruction at this level should also focus on developing children's interest in mathematics.

- Students in grades 6 through 8 are forming conclusions about their mathematical abilities, interest, and motivation that will influence how they approach mathematics in later years. Instruction at this level should build on their emerging capabilities to think hypothetically, comprehend cause and effect, and reason in both concrete and abstract terms. Algebra and geometry form a large part of the recommended curriculum during these years.

An important key to developmentally appropriate mathematics instruction, at any age or grade level, is achieving balance between teaching for conceptual understanding and teaching for procedural fluency. When students learn procedures without meaning, they are only memorizing discrete pieces of information that are difficult for them to remember. Students should develop an understanding of the concepts they are studying before they apply these ideas to procedural strategies.

### Good Teaching Is Key

Of course, effective mathematics instruction begins with effective teaching. No lesson, no matter how well planned, can be successful if the elements of effective teaching are not in place. Grouws (2004) discusses the instructional practices that research has shown to have a positive impact on student learning and then mentions the role of the teacher:

*The quality of the implementation of a teaching practice also greatly influences its impact on student learning. The value of using manipulative materials to investigate a concept, for example, depends not only on whether manipulatives are used, but also on how they are used with the students. Similarly, small-group instruction will benefit students only if the teacher knows when and how to use this teaching practice.*

In addition, to effectively develop students' mathematical skills, teachers must be effective overall. They must exhibit good classroom management skills, especially in classrooms using differentiated instruction; actively engage their students; and make efficient use of instructional time. A mathematics lesson cannot succeed if the other elements of teaching—classroom management, a logical progression of lessons, an effective use of assessment, and time management—are not in place.

### An Effective Mathematics Environment

There are some specific teacher behaviors that "matter" in the teaching of mathematics. In effective classrooms, teachers:

- *Demonstrate acceptance of students' divergent ideas.* They challenge students to think more deeply about the problems they are solving and ask them to explain the solutions.

Such an approach also helps students develop confidence in their own abilities to do mathematics and gain an even firmer grasp of key concepts and processes.

- *Influence learning by posing challenging and interesting questions.* Teachers should present questions that stimulate students' curiosity and encourage them to investigate further. The questions should encourage students to rely on themselves and their peers for ideas about mathematics and problem-solving.
- *Project a positive attitude about mathematics and about students' ability to "do" mathematics.* This includes demonstrating enthusiasm for the content as well as a belief that all students are capable of learning the material, with lessons designed to encourage curiosity, interest, and skill-building.

Certain instructional characteristics also are associated with effective math-

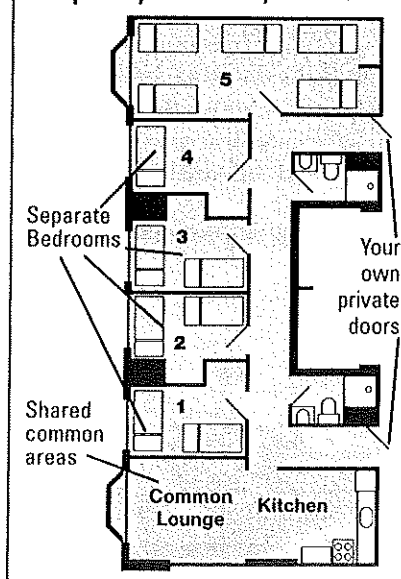
ematics instruction. By integrating the following approaches into classroom instruction, teachers can promote both student learning and motivation:

- *Students are actively engaged in doing mathematics.* They should not be sitting back watching others students solve problems.
- *Students are solving challenging problems.* Mathematics is a stimulating and interesting field generating new knowledge every day, and students should be exposed to this excitement and challenge, using real-world examples when possible.
- *Interdisciplinary connections and examples are used to teach mathematics.* For example, using literature as a springboard for mathematical investigation is a useful way to introduce authentic problem-solving situations that may have "messy" results. This engages students in connecting the language of mathematical ideas with numerical representations and

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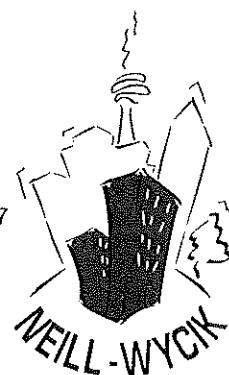
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develops important skills that support students' abilities to solve word problems.

- *Students are sharing their mathematical ideas while working in pairs and groups.* Research shows that students who work in groups on problems, assignments, and other mathematical investigations display increased achievement. Such opportunities appeal to the social nature of most children, while thinking through problems collaboratively makes it less likely that a student will get caught in a procedural dead end.
- *Students are provided with a variety of opportunities to communicate mathematically.* During a lesson, students should have many opportunities to communicate their ideas. They may draw a picture to represent their ideas or write them in mathematics journals. Whole-class discussions should provide opportunities to hear about and perhaps challenge other students' ideas in an environment of respect and understanding.
- *Students are using manipulatives and other tools.* The long-term use of mathematics manipulatives is positively related to student achievement and attitudes about mathematics. It is not enough, however, to simply provide students with manipulatives; they must be taught how to use these materials. Several steps can be taken to ensure students benefit from a lesson involving manipulatives. First, the teacher should use manipulatives that support the lesson's objectives. Next, before allowing students to handle the materials, the teacher should demonstrate how to use the manipulatives and the procedures for handling them. And finally, the lesson design should encourage the active participation of all students (Ross and Kurtz 1993).

### An Ongoing Dialogue

Conversations about math instruction will continue. Parents, educators, policymakers, and future employers are all concerned about what—and how well—our students learn mathematics.

In recognition of the importance of the topic, a National Mathematics Advisory Panel was recently established within the U.S. Department of Education. Its charge is "to foster greater knowledge of and improved performance in mathematics among American students ... with respect to the conduct, evaluation, and effective use of the results of research relating to proven-effective and evidence-based mathematics instruction" (Bush 2006). Members of the panel have been assigned to four task forces focused on critical areas of mathematics instruction: learning processes, conceptual knowledge and skills, instructional practices and materials, and teachers and teacher education (National Mathematics Advisory Panel 2007).

For teachers and others responsible for ensuring our students receive the best possible mathematics education, ongoing efforts to stay informed about current research findings will be as important as the frameworks provided by state standards. As principal, your role in ensuring that happens in your school is critical. □

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### WEB RESOURCES

Information provided on the NCTM Web site includes the council's focal points by grade as well as questions and answers about the focal points. [www.nctm.org/focalpoints.aspx?ekmsel=c580fa7b\\_10\\_48\\_btnlink](http://www.nctm.org/focalpoints.aspx?ekmsel=c580fa7b_10_48_btnlink)

"Mathematical Understanding: An Introduction," a chapter from the book *How Students Learn: History, Mathematics, and Science in the Classroom*, can be accessed at the National Academies Press Web site. [www.nap.edu/catalog.php?record\\_id=10126#toc](http://www.nap.edu/catalog.php?record_id=10126#toc)

The K-12 Mathematics Curriculum Center, funded by the National Science Foundation, provides information on textbook adoption and a downloadable guide to several mathematics curriculum programs. [www2.edc.org/mcc/default.asp](http://www2.edc.org/mcc/default.asp)

